

$$\begin{aligned}
 6. \quad & w + x + y - z = 0 \\
 & 2w - 3x - y + 2z = 1 \\
 & -w + x + 2y + 0z = 5 \\
 & w + 0x + 0y + 3z = 9
 \end{aligned}$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & -1 & 0 \\ 2 & -3 & -1 & 2 & 1 \\ -1 & 1 & 2 & 0 & 5 \\ 1 & 0 & 0 & 3 & 9 \end{array} \right]$$

$$2F_1 - F_2 \rightarrow F_1$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & -1 & 0 \\ 0 & 5 & 3 & -4 & -1 \\ -1 & 1 & 2 & 0 & 5 \\ 1 & 0 & 0 & 3 & 9 \end{array} \right]$$

$$\begin{aligned}
 2(1) - (-3) &= 2+3=5 \\
 2(1) - (-1) &= 2+1=3 \\
 2(-1) - 2 &= -2-2=-4 \\
 2(0) - 1 &= -1
 \end{aligned}$$

$$\frac{F_2}{5} \rightarrow F_2$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & -1 & 0 \\ 0 & 1 & 3/5 & -4/5 & -1/5 \\ -1 & 1 & 2 & 0 & 5 \\ 1 & 0 & 0 & 3 & 9 \end{array} \right]$$

$$F_3 + F_1 \rightarrow F_3$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & -1 & 0 \\ 0 & 1 & 3/5 & -4/5 & -1/5 \\ 0 & 2 & 3 & -1 & 5 \\ 1 & 0 & 0 & 3 & 9 \end{array} \right]$$

$$\begin{aligned}
 -1+1 &= 0 \\
 1+1 &= 2 \\
 2+1 &= 3 \\
 0-1 &= -1
 \end{aligned}$$

$$5+0=5$$

$$2F_2 - F_3 \rightarrow F_3$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & -1 & 0 \\ 0 & 1 & 3/5 & -4/5 & -1/5 \\ 0 & 0 & -9/5 & -3/5 & -23/5 \\ 1 & 0 & 0 & 3 & 9 \end{array} \right]$$

$$2(3/5) - 2 = \frac{6}{5} - \frac{10}{5} = -\frac{4}{5}$$

$$2(-4/5) - (-1) = -\frac{8}{5} + 1 = -\frac{8}{5} + \frac{5}{5} = -\frac{3}{5}$$

$$2(-1/5) - 5 = -\frac{2}{5} - \frac{25}{5} = -\frac{27}{5}$$

$$-\frac{5}{9} F_3 \rightarrow F_3$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & -1 & 0 \\ 0 & 1 & 3/5 & -4/5 & -1/5 \\ 0 & 0 & 1 & 1/3 & 3 \\ -1 & 0 & 0 & 3 & 9 \end{array} \right]$$

$$-\frac{5}{9}(-\frac{4}{5}) = \frac{1}{3}$$

$$-\frac{5}{9}(-\frac{3}{5}) = 3$$

$$F_4 - F_1 \rightarrow F_4$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & -1 & 0 \\ 0 & 1 & 3/5 & -4/5 & -1/5 \\ 0 & 0 & 1 & 1/3 & 3 \\ 0 & -1 & -1 & 4 & 9 \end{array} \right]$$

$$0-1=-1$$

$$9-0=9$$

$$0-1=-1$$

$$3-(-1)=3+1=4$$

$$F_4 + F_2 \rightarrow F_4$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & -1 & 0 \\ 0 & 1 & 3/5 & -4/5 & -1/5 \\ 0 & 0 & 1 & 1/3 & 3 \\ 0 & 0 & 1/3 & 14/5 & 49/5 \end{array} \right]$$

$$-1 + \frac{2}{3} = -\frac{3}{3} + \frac{2}{3} = -\frac{1}{3}$$

$$4 - \frac{4}{5} = \frac{20}{5} - \frac{4}{5} = \frac{16}{5}$$

$$9 - \frac{1}{5} = \frac{45}{5} - \frac{1}{5} = \frac{44}{5}$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & -1 & 0 \\ 0 & 1 & \frac{1}{5} & -\frac{1}{5} & -\frac{1}{5} \\ 0 & 0 & 1 & \frac{1}{5} & 3 \\ 0 & 0 & -\frac{1}{5} & \frac{1}{5} & \frac{4}{5} \end{array} \right]$$

$$\frac{2}{5}F_3 + F_4 \rightarrow F_4$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & -1 & 0 \\ 0 & 1 & \frac{1}{5} & -\frac{1}{5} & -\frac{1}{5} \\ 0 & 0 & 1 & \frac{1}{5} & 3 \\ 0 & 0 & 0 & \frac{10}{3} & \frac{50}{3} \end{array} \right]$$

$$\frac{1}{5}\left(\frac{1}{3}\right) + \frac{16}{5} = \frac{2}{15} + \frac{16}{5} = \frac{10}{3}$$

$$\frac{2}{5}(3) + \frac{44}{5} = \frac{6}{5} + \frac{44}{5} = \frac{50}{5}$$

$$\frac{10}{3}z = \frac{50}{5}$$

$$z = \frac{50}{5} \left(\frac{3}{10} \right)$$

$$z = \frac{15}{5} = 3$$

$$y + \frac{1}{3}z = 3$$

$$y + \frac{1}{3}(3) = 3$$

$$y = 3 - 1$$

$$y = 2$$

$$x + \frac{2}{5}y - \frac{4}{5}z = -\frac{1}{5}$$

$$x = -\frac{1}{5} - \frac{2}{5}y + \frac{4}{5}z$$

$$x = -\frac{1}{5} - \frac{2}{5}(2) + \frac{4}{5}(3)$$

$$x = -\frac{1}{5} - \frac{6}{5} + \frac{12}{5}$$

$$x = \frac{-1-6+12}{5}$$

$$x = \frac{-7+12}{5}$$

$$x = \frac{5}{5} = 1$$

$$w + x + y - z = 0$$

$$w = z - x - y$$

$$w = 3 - 1 - 2$$

$$w = 0$$

$$\begin{aligned}
 7. \quad & 3x + 2y + z - 10w = 0 \\
 & 2x - 3y - z - 2w = 0 \\
 & 5x - y - 2z + 4w = 0
 \end{aligned}$$

$$\left[\begin{array}{cccc|c} 3 & 2 & 1 & -10 & 0 \\ 2 & -3 & -1 & -2 & 0 \\ 5 & -1 & -2 & 4 & 0 \end{array} \right]$$

$$\frac{F_1}{3} \rightarrow F_1$$

$$\left[\begin{array}{cccc|c} 1 & 2/3 & 1/3 & -10/3 & 0 \\ 2 & -3 & -1 & -2 & 0 \\ 5 & -1 & -2 & 4 & 0 \end{array} \right]$$

$$2F_1 - F_2 \rightarrow F_2$$

$$\left[\begin{array}{cccc|c} 1 & 2/3 & 1/3 & -10/3 & 0 \\ 0 & 13/3 & 5/3 & -14/3 & 0 \\ 5 & -1 & -2 & 4 & 0 \end{array} \right]$$

$$2\left(\frac{2}{3}\right) - (-3) = \frac{4}{3} + 9 = \frac{13}{3}$$

$$2\left(\frac{1}{3}\right) - (-1) = \frac{2}{3} + 1 = \frac{5}{3}$$

$$\frac{5}{13} F_2 \rightarrow F_2$$

$$2\left(-\frac{10}{3}\right) - (-2) = -\frac{20}{3} + \frac{2}{3} = -\frac{18}{3}$$

$$\left[\begin{array}{cccc|c} 1 & 2/3 & 1/3 & -10/3 & 0 \\ 0 & 1 & 5/13 & -14/13 & 0 \\ 5 & -1 & -2 & 4 & 0 \end{array} \right]$$

$$\frac{5}{13}\left(\frac{5}{3}\right) = \frac{25}{39}$$

$$\frac{5}{13}\left(-\frac{14}{3}\right) = -\frac{70}{39}$$

$$5F_1 - F_3 \rightarrow F_3$$

$$\left[\begin{array}{cccc|c} 1 & 2/3 & 1/3 & -10/3 & 0 \\ 0 & 1 & 5/13 & -14/13 & 0 \\ 0 & 13/3 & 11/3 & -62/3 & 0 \end{array} \right]$$

$$5\left(\frac{2}{3}\right) - (-1) = \frac{10}{3} + 1 = \frac{13}{3}$$

$$5\left(\frac{1}{3}\right) - (-2) = \frac{5}{3} + 2 = \frac{11}{3}$$

$$3F_3 \rightarrow F_3$$

$$\begin{aligned}
 5\left(-\frac{10}{3}\right) \cdot 4 &= -\frac{50}{3} - \frac{12}{3} \\
 &= -\frac{62}{3}
 \end{aligned}$$

$$\left[\begin{array}{cccc|c} 1 & 2/3 & 1/3 & -10/3 & 0 \\ 0 & 1 & 5/13 & -14/13 & 0 \\ 0 & 13 & 11 & -62 & 0 \end{array} \right]$$

$$\left[\begin{array}{cccc|c} 1 & 2/3 & 1/3 & -10/3 & 0 \\ 0 & 1 & 5/13 & -14/13 & 0 \\ 0 & 13 & 11 & -62 & 0 \end{array} \right]$$

$$13F_2 - F_3 \rightarrow F_3$$

$$\left[\begin{array}{cccc|c} 1 & 2/3 & 1/3 & -10/3 & 0 \\ 0 & 1 & 5/13 & -14/13 & 0 \\ 0 & 0 & -6 & 48 & 0 \end{array} \right]$$

$$13\left(\frac{5}{13}\right) - 11 = 5 - 11 = -6$$

$$13\left(-\frac{14}{13}\right) - (-62) = -14 + 62 = 48$$

$$-6z + 48w = 0 \quad w = c$$

$$-6z = -48c$$

$$z = \frac{-48c}{-6}$$

$$z = 8c$$

$$y + \frac{5}{13}z - \frac{14}{13}w = 0$$

$$y = \frac{14}{13}c - \frac{5}{13}(8c)$$

$$y = \frac{14}{13}c - \frac{40}{13}c$$

$$y = -\frac{26}{13}c$$

$$x + \frac{2}{3}y + \frac{1}{3}z - \frac{10}{3}w = 0$$

$$y = -2c$$

$$x = \frac{10}{3}w - \frac{1}{3}z - \frac{2}{3}y$$

$$x = \frac{10}{3}c - \frac{1}{3}(8c) - \frac{2}{3}(-2c) \quad \begin{matrix} 0 \\ 0 \\ 0 \end{matrix}$$

$$x = \frac{10}{3}c - \frac{8}{3}c + \frac{4}{3}c$$

$$x = 2c$$

$$x = 2c$$

$$y = -\frac{26}{13}c$$

$$z = 8c$$

$$w = c$$

$$P//(x, y, z, w) = (2c, -2c, 8c, c)$$

$$c \in \mathbb{R}$$

$$\begin{array}{rcl}
 & w & x & y & z \\
 8. & & +x & +2y & = 6 \\
 & w & +x & & +2z = 8 \\
 & 2w & +x & +y & +z = 10 \\
 & & & +2y & -3z = 0
 \end{array}$$

$$\left[\begin{array}{cccc|c} 0 & 1 & 2 & 0 & 6 \\ 1 & 1 & 0 & 2 & 8 \\ 2 & 1 & 1 & 1 & 10 \\ 1 & 0 & 2 & -3 & 0 \end{array} \right]$$

$$\begin{array}{l}
 8. \quad x + 2y + 2z - 10w = 0 \\
 6x + 5y - 2z - 4w = 0 \\
 2x - y + 16z - 2w = 0
 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 2 & 2 & -10 & 0 \\ 6 & 5 & -2 & -4 & 0 \\ 2 & -1 & 16 & -2 & 0 \end{array} \right]$$

$$6F_1 - F_2 \rightarrow F_2$$

$$\left[\begin{array}{cccc|c} 1 & 2 & 2 & -10 & 0 \\ 0 & 7 & 14 & -56 & 0 \\ 2 & -1 & 16 & -2 & 0 \end{array} \right]$$

$$6(2) - 5 = 12 - 5 = 7$$

$$6(2) - (-4) = 12 + 4 = 16$$

$$6(-10) - (-2) = -60 + 2 = -58$$

$$\frac{F_2}{7} \rightarrow F_2$$

$$\left[\begin{array}{cccc|c} 1 & 2 & 2 & -10 & 0 \\ 0 & 1 & 2 & -56/7 & 0 \\ 2 & -1 & 16 & -2 & 0 \end{array} \right]$$

$$2F_1 - F_3 \rightarrow F_3$$

$$\left[\begin{array}{cccc|c} 1 & 2 & 2 & -10 & 0 \\ 0 & 1 & 2 & -56/7 & 0 \\ 0 & 5 & -12 & -18 & 0 \end{array} \right]$$

$$2(2) - (-1) = 4 + 1 = 5$$

$$2(2) - 16 = 4 - 16 = -12$$

$$2(-10) - (-2) = -20 + 2 = -18$$

$$5F_2 - F_3 \rightarrow F_3$$

$$\left[\begin{array}{cccc|c} 1 & 2 & 2 & -10 & 0 \\ 0 & 1 & 2 & -56/7 & 0 \\ 0 & 0 & 22 & -22 & 0 \end{array} \right]$$

$$5(2) - (-12) = 10 + 12 = 22$$

$$5(-56/7) - (-18) = -40 + 18 = -22$$

CER

$$w = C$$

$$22z - 22w = 0$$

$$22z - 22C = 0$$

$$22z = 22C$$

$$z = C$$

$$y + 2z - \frac{56}{7}w = 0$$

$$y = \frac{56}{7}w - 2z$$

$$y = \frac{56}{7}C - \frac{14}{7}C$$

$$y = \frac{42}{7}C$$

$$y = 6C$$

$$x + 2y + 2z - 10w = 0$$

$$x = 10w - 2y - 2z$$

$$x = 10C - 2(6C) - 2C$$

$$x = 10C - 12C - 2C$$

$$x = -4C$$

9. $2x - 3y + 4z = 1$ $\xleftrightarrow[\text{a conveniencia}]{\text{ordenamos}}$ $\begin{cases} x - y + z = 5 \\ 2x - 3y + 4z = 1 \end{cases}$

$$\left[\begin{array}{ccc|c} 1 & -1 & -1 & 5 \\ 2 & -3 & 4 & 1 \end{array} \right]$$

$$2F_1 - F_2 \rightarrow F_2$$

$$\left[\begin{array}{ccc|c} 1 & -1 & -1 & 5 \\ 0 & 1 & -6 & 9 \end{array} \right]$$

$$z = t \quad t \in \mathbb{R}$$

$$y - 6z = 9$$

$$y = 9 - 6t$$

$$2(-1) - (-3) = -2 + 3 = 1$$

$$2(-1) - 4 = -2 - 4 = -6$$

$$2(5) - 1 = 10 - 1 = 9$$

$$x - y - z = 5$$

$$x = y + z + 5$$

$$x = 9 - 6t + t + 5$$

$$x = 14 - 5t$$

$$(x, y, z) = (14 - 5t, 9 - 6t, t) \quad , \quad t \in \mathbb{R}$$

10. $\begin{cases} 3x + 2y - z = 7 \\ x - 4y + 2z = 0 \end{cases} \xleftrightarrow[\text{A conveniencia}]{\text{Resolvemos...}}$ $\begin{cases} x - 4y + 2z = 0 \\ 3x + 2y - z = 7 \end{cases}$

$$\left[\begin{array}{ccc|c} 1 & -4 & 2 & 0 \\ 3 & 2 & -1 & 7 \end{array} \right]$$

$$3F_1 - F_2 \rightarrow F_2$$

$$\left[\begin{array}{ccc|c} 1 & -4 & 2 & 0 \\ 0 & -14 & 7 & -7 \end{array} \right]$$

$$3(-4) - 2 = -12 - 2 = -14$$

$$3(2) - (-1) = 6 + 1 = 7$$

$$3(0) - 7 = -7$$

$$-14y + 7z = -7$$

$$z = t, \quad t \in \mathbb{R}$$

$$-14y = -7 - 7t$$

$$y = \frac{-7(1+t)}{-14} = \frac{1+t}{2}$$

$$y = \frac{1+t}{2}$$

$$x - 4y + 2z = 0$$

$$x = 4y - 2z$$

$$x = 4\left(\frac{1+t}{2}\right) - 2t$$

$$x = 2 + 2t - 2t$$

$$x = 2$$

$$(x, y, z) = \left(2, \frac{1+t}{2}, t\right)$$

Donde

$$t \in \mathbb{R}$$